The determination of W_g is the next problem. Consider Eq. (7). As has been shown,[6] K_r is negligible at all temperatures for Ge. The measurements of Ukhanov[51] on the optical absorption of Si and Ge at elevated temperatures were employed to calculate K_r. This calculation shows that K_r is also negligible (i.e., less than 2% of total K) for Si at all temperatures. The K_e term in Eq. (7) becomes important only at the very highest temperatures when the number of thermally generated carriers becomes very large. This term is responsible for the upturn in the K results shown in Figs. 2 and 3 above 800°K. Thus K_g is the only significant term in Eq. (7) for $T<800$°K in Si and Ge. Therefore, the total measured thermal conductivity is $K=W_g^{-1}$ for $T<800$°K. Figures 10 and 11 show the plots of $(W-W_I)T^{-1}$ versus T for Si and Ge. The linear approximation predicted by Eq. (11) is satisfactory for $T\geq 0.5\theta$ until at high temperatures the K_e term becomes important. The dashed extrapolations in Figs. 10 and 11 show what the behavior would be if only the lattice thermal conductivity K_g were present.

The linear portions of the curves in Figs. 10 and 11 yield the following empirical expressions for W_g:

$$\begin{aligned}\text{Ge:}\quad W_g&=(3.95\times10^{-3}T+3.38\times10^{-6}T^2+0.17)\ \text{cm deg/W},\\ \text{Si:}\quad W_g&=(1.56\times10^{-3}T+1.65\times10^{-6}T^2+0.03)\ \text{cm deg/W}.\end{aligned}\tag{12}$$

The first term in these equations yields an experimental value of B_U which agrees with that estimated from Eq. (5) to within a factor of two for both Si and Ge. Such agreement is quite satisfactory. If the very approximate theory used in obtaining Eq. (11) is taken at face value, the results in Eq. (12) for B_H show that the ratio τ_H/τ_U is 3 for Ge and 1 for Si at $T=\theta$. This ratio becomes smaller at higher temperatures. Notice that we have chosen to interpret the observation that K_g for Si and Ge at high temperatures decreases faster than T^{-1} as a sign that 4-phonon processes are taking place. A similar conclusion has been arrived at by Stuckes[19] and by Steigmeier and Kudman[52] for various IV and III-V semiconductors. The result that the 3-phonon and 4-phonon relaxation times, τ_H and τ_U, are nearly equal at $T=\theta$ should, however, be greeted with some caution. The main purpose of the plots in Figs. 10 and 11 is to allow one to make a reasonable extrapolation of K_g in the temperature region where K_e is appreciable.

This extrapolation of K_g from Figs. 10 and 11 has been replotted as the dashed curve in Figs. 2 and 3. The difference between this curve and the experimental results is, from Eq. (7), just K_e. These experimental values of K_e for Si and Ge are plotted in Figs. 12 and 13.

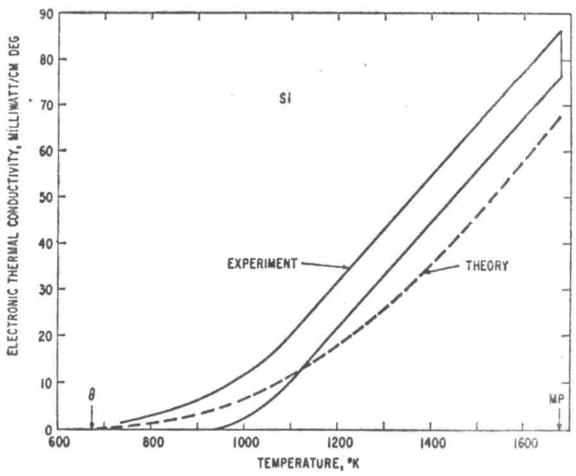

FIG. 12. Comparison of the experimental and theoretical electronic thermal conductivity, K_e versus T, for Si.

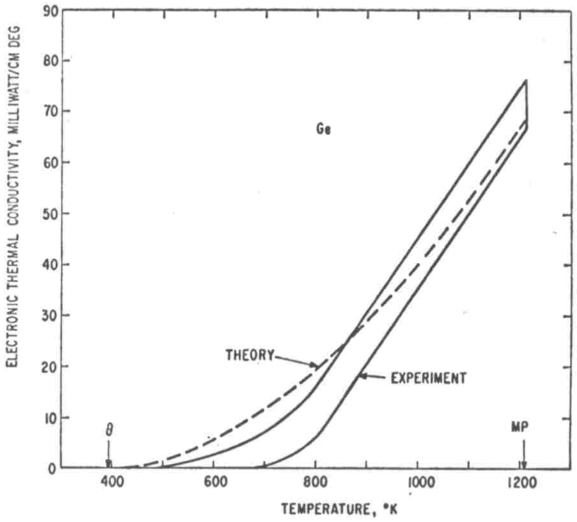

FIG. 13. Comparison of the experimental and theoretical electronic thermal conductivity, K_e versus T, for Ge.

Electronic Thermal Conductivity

The experimental results for K_e can be compared with the theory that has been developed by Davydov and Shmushkevitch,[53] by Price,[54] and by Drabble and Goldsmid.[55] Their results for K_e can be written as the sum of an electronic polar contribution K_{ep}, and an electronic bipolar part K_{eb}.

$$K_e=K_{ep}+K_{eb},$$

where

$$\begin{aligned}K_{ep}&=2\left[\frac{k}{e}\right]^2\sigma T,\\ K_{eb}&=\frac{b}{(1+b)^2}\left[\frac{E_G}{kT}+4\right]^2\left[\frac{k}{e}\right]^2\sigma T.\end{aligned}\tag{13}$$

[51] Yu. I. Ukhanov, Fiz. Tverd. Tela **3**, 2105 (1961) [English transl.: Soviet Phys.—Solid State **3**, 1529 (1962)].

[52] E. F. Steigmeier and I. Kudman, Phys. Rev. **132**, 508 (1963).
[53] B. Davydov and I. Shmushkevitch, Uspekhi Fiz. Nauk USSR 24, 21 (1940).
[54] P. J. Price, Phil. Mag. **46**, 1252 (1955).
[55] J. R. Drabble and H. J. Goldsmid, *Thermal Conduction in Semiconductors* (Pergamon Press, Inc., London, 1961), pp. 104–119.